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Models for Hadron-Hadron Scattering at High Energies And Rising Total Cross-Sections

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ABSTRACT

The recent results of rising total cross-sections for $\pi^{\pm}p$, $K^{\pm}p$, pp and $\bar{p}p$ scattering are analyzed by two simple analytic models for high energy forward scattering, which are derived from analyticity, crossing symmetry and the unitarity constraints of the rigorous results. The numerical fit to the data for $p_L \geq 10$ GeV/c suggests strongly that the crossing-odd amplitude may not be negligible at high energies.

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The recent results[1] of an experiment at the Fermi National Accelerator Laboratory prove that the total cross sections for pp, πp and Kp scattering all show indeed a remarkable rise in the range of $50 \leq p_L \leq 200$ GeV/c, thus confirming beautifully the previous observations[2] made for pp at the CERN ISR. Already a number of theoretical models[3,4] have been put forth to explain such a rise in σ_{pp} seen at the ISR. Among them, the works of Ref. 4, in particular, indicate the existence of a $\ln^2 s$ term in σ_T. However, in actuality these works predict either too fast an increase of σ_T or too rapid a decrease in $\sigma_{\bar{p}p}$ when compared with the Fermilab data. Thus, it will be interesting to develop a theoretical framework that can explain the seemingly general behavior of all σ_T, as well as the ratio $\alpha \equiv \mathrm{ReF(s)/ImF(s)}$.

In this paper, we propose some simple analytic parametrizations for high energy hadron-hadron scattering, which are derived from the "quasi-local" relations of analyticity and crossing symmetry with the constraints coming from the rigorous studies of analyticity, unitarity and positivity. Such a quasi-local relation has been suggested some time ago[5] and several authors[6] have advocated it recently again.

As it has been emphasized by many people,[7] most of the asymptotic statements in the literature involve an <u>additional</u> tacit assumption that the crossing-odd amplitude becomes negligible at high energies. But one can construct an example in which the crossing-odd amplitude can even grow in energy and yet is perfectly compatible with all the results of rigorous studies.[8] We keep such option in mind in developing the basic analyticity relations and their solutions.

We can then derive from the anlyticity relation in the form of the Sommerfeld-Watson-Regge representation that

$$\mathrm{Re}\left\{F_+(s,t)/s\right\}=\left\{\frac{\pi}{2}\frac{\partial}{\partial \ln s}+\frac{1}{3}\left(\frac{\pi}{2}\frac{\partial}{\partial \ln s}\right)^3+\frac{2}{15}\left(\frac{\pi}{2}\frac{\partial}{\partial \ln s}\right)^5+\cdots\right\}\mathrm{Im}\left\{F_+(s,t)/s\right\} \quad (1)$$

$$\frac{\pi}{2}\frac{\partial}{\partial \ln s}\,\mathrm{Re}\left\{F_-(s,t)/s\right\}=-\left\{1-\frac{1}{3}\left(\frac{\pi}{2}\frac{\partial}{\partial \ln s}\right)^2-\frac{1}{45}\left(\frac{\pi}{2}\frac{\partial}{\partial \ln s}\right)^4-\cdots\right\}\mathrm{Im}\left\{F_-(s,t)/s\right\} \quad (2)$$

where the crossing-even and odd amplitudes are defined as $F_\pm=\frac{1}{2}(F_{AB}\pm F_{\bar{A}B})$ and normalized so that $s\sigma_T(s) = \mathrm{Im}F(s,o)$. In order to derive these,[9] we have assumed that the asymptotic behavior of $F_\pm$ is controlled by the right-most singularities of $A(\ell\,,t)$ whose position is one in the forward direction for the reasons explained above. This is certainly possible even if the analytic continuation of the Froissart-Gribov relation down to $\ell=1$ may not coincide with the physical p-wave amplitude.[10]

The relations (1) and (2) are "quasi-local" due to the inherent asymptotic nature. They imply the convenience of using $\ln s$ as a natural variable in the high-energy region. Then together with de facto rise of total cross-sections, they are designed to describe the situations as well in which the singularities of $A(\ell\,,t)$ in the complex ℓ-plane are not necessarily just simple poles. In addition, these quasi-local relations have a number of interesting features: (a) F_+ is predominantly imaginary at high energies while F_- can be real, which is a well-known consequence of the rigorous studies[8]; (b) if $|F_+|\propto s(\ln s)^{\beta_+}$ with $\beta_+\le 2$ then we get from (1) that $\alpha_+ = \mathrm{Re}F_+/\mathrm{Im}F_+ \to \frac{\pi\beta_+}{2}(\ln s)^{-1}$, again a well-known result[11]; (c) if, in addition $|F_-|\propto s(\ln s)^{\beta_-}$ such that [8] $\beta_- -1 \le \beta_+/2$, then it follows from (1) and (2) that $|\sigma_{\bar{A}B} - \sigma_{AB}| < (\sigma_{\bar{A}B} + \sigma_{AB})/\ln s$, the rigorous form of the Pomeranchuk theorem.[12]

Denoting $F_{\pm}(s,o) \equiv F_{\pm}(s)$ and allowing that $A(\ell, t)$ can possess harder singularities than simple poles, we may write

$$\mathrm{Im}\left\{F_+(s)/s\right\} = A_+ + B_+ \ln s + C_+ \ln^2 s + C_R s^{-\frac{1}{2}} \tag{3}$$

$$\mathrm{Im}\left\{F_-(s)/s\right\} = B_- + \pi C_- \ln s - C_R s^{-\frac{1}{2}} \tag{4}$$

which are compatible with the unitarity bounds of the rigorous studies. Here the C_R terms are introduced to represent the over-all contributions of the exchange-degenerate Reggeons. Also it is understood that $\ln s = \ln s/\mathrm{GeV}^2$ at this point but the real scale will be determined by the fit to the experimental data. Then the <u>exact</u> analytic solutions of $\mathrm{Re}F_{\pm}$ can be obtained from (1) and (2);

$$\mathrm{Re}\left\{F_+(s)/s\right\} = (\pi/2)\left\{B_+ + 2C_+ \ln s - (2/\pi) C_R s^{-\frac{1}{2}}\right\} \tag{5}$$

$$\mathrm{Re}\left\{F_-(s)/s\right\} = -(2/\pi)\left\{A_- + B_- \ln s + (\pi/2) C_- \ln^2 s + (\pi/2) C_R s^{-\frac{1}{2}}\right\} \tag{6}$$

Thus we end up with the analytic parametrization[13]

$$F_+(s)/is = \sigma_0 + C_+(\ln^2 s/s_+ - i\pi \ln s/s_+) + C_R(i+1)s^{-\frac{1}{2}} \tag{7}$$

$$F_-(s)/is = iC_-(\ln^2 s/s_- - i\pi \ln s/s_-) + C_R(i-1)s^{-\frac{1}{2}} \tag{8}$$

Depending on the absence of the C_- and C_R terms, we end up with different physical pictures: (a) M-model. This is the case of $C_- = 0$ and is the conventional and perhaps the most "moderate" line of pictures, e.g., the works of Cheng, Walker and Wu, and Bourrely and Fischer. But they give in general a faster increase of σ_T than the Fermilab data indicate; (b) I-model. This is the case of $C_R = 0$ so that even ρ does not contribute to F_- . This may appear to a traditional Reggeist improbable or even "immoral". The parametrization of Lukaszuk and Nicolescu is a very special case of this picture, i.e., $C_+ = C_-$ and $s_+ = s_-$, which, however, gives a faster decrease of $\sigma_{\bar{p}p}$ than the actual data show; (c) SI-model. This is the case of $C_- \neq 0$ and $C_R \neq 0$, i.e., the full structure of (7)and (8).

As it seems that the reality may lie somewhere between the full I-model and full SI-model, we have made a complete χ^2-fit (Fig. 1) to all available data[14] of σ_T with the two analytic parametrizations for πp, Kp and pp scattering above $p_L \geq 10$ GeV/c. In the case of πp scattering, we have incorporated in addition the 18 data points[14] of the charge-exchange differential cross-sections at $t = 0$ (Fig. 3). The ratios $\alpha = \mathrm{ReF}(s)/\mathrm{ImF}(s)$ are predicted and compared with the existing data (Fig. 4-6). The parameters corresponding to the best fit in each picture are summarized in Table 1 along with the χ^2-value.

One can see that the over-all fit in both parametrizations is quite remarkable in all the cases. They give a similar fit to σ_T (Fig. 1) and in the case of σ_{pp} they are practically identical. Note from Fig. 2 that both the I-model and the SI-model can fit $\Delta\sigma_T$ equally well except the case of πp. Here it seems that we are dealing with two systemically different sets of data. From the characteristic curvature of the $\ln(\Delta\sigma_T)$ vs $\ln p_L$ plot, however, the SI-model appears to be favored by the Fermilab data while the Serpukhov data are fitted better by the I-model parametrization. Experimental clarification of the discrepancy can help to distinguish between the two models. On the other hand, we see from Fig. 3 that they both give an equally excellent fit to $(d\sigma/dt)$ at $t = 0$ for $\pi^- p \to \pi^0 n$. The fact that the I-model, which does not contain a ρ contribution, can fit $(d\sigma/dt)_0$ and $\Delta\sigma_T$ equally well is simply amazing. Also α_{pp} and $\alpha_{\bar{p}p}$ can be described more or less to the same fairness by both parametrizations (Fig. 4). The only difficulty is in $\alpha_{\pi^- p}$ around $p_L = 58$ GeV/c. (Fig. 5). If this datum is indeed correct, then all models with $\sigma_T \propto \ln^2 s$ including the M-model will have a serious problem to explain this. The best value that we can come up with is $\alpha_{\pi^- p}$ ($p_L = 58$ GeV/c) $\cong -0.01$ instead of -0.08 ± 0.03.

To conclude, we emphasize that while the actual data at this point in time do not favor any one unique parametrization, they are nevertheless compatible with $C_- \neq 0$, thus suggesting various different physical consequences: (1) The crossing-odd amplitude can be predominantly real at high energies, making both the real and imaginary parts of the forward amplitudes F_{AB} and $F_{\bar{A}B}$ to grow like $s \ln^2 s$. (2) if $C_- > 0$, $\sigma_{\bar{A}B}$ and σ_{AB} must cross at some $s \geq s_-$ and $\alpha_{AB} \to -C_-/C_+$, a negative number, after having gone through two zeros; (3) if $C_- < 0$, then there is no cross-over in the total cross sections. But $\Delta\sigma_T$ reaches a minimum at $s = (C_R/2\pi C_-)^2$ and $\alpha_{AB} \to -C_-/C_+$, a positive limit after one sign change. From the χ^2-fit, we see that the Kp and pp interactions are of the type (2) in either pictures, while the πp interactions again distinguish between the two pictures, i.e., the I-model is of the type (2) and the SI-model is of the type (3). In particular, there is a universal cross-over in I-model for all three reactions at $p_L \cong 500$ GeV/c. In the case of SI-model, the crossing is delayed until after $p_L \gtrsim 1{,}000$ GeV/c for Kp and pp, while $(\Delta\sigma_T)_{\pi p}$ has a minimum at

$p_L \cong 670$ GeV/c.

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[9] Eq. (2) is the more convenient form to deal with the possibility that $\mathrm{Im}\{F_-(s)/s\} \propto (\ln s)^{\beta_-}$ with $0 < \beta_- \leq 1$. For a decreasing power form of $\mathrm{Im}\{F_-(s)/s\}$, it reduces to the result of Ref. 6. Details of the derivation as well the expanded analysis will be published elsewhere.

[10] Then the p-wave amplitude should be given from the outset by the subtraction term and the contour of the Sommerfeld-Watson-Regge transformation should be chosen so as not to include the point $\ell = 1$. We thank Alan R. White for the clarifying discussions on this technicality.

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[12] R. J. Eden, Phys. Rev. Letters 16, 39 (1966); T. Kinoshita, Perspective in Modern Physics, Interscience Publ., New York (1966).

[13] We have set $\sigma_0 = A_+ - C_+ \ln^2 s_+$, $B_+ = -2C_+ \ln s_+$, $B_- = -\pi C_- \ln s_-$ and $A_- = \frac{\pi}{2} C_- \ln^2 s_-$. In general we can allow one more parameter $\sigma_1 = \frac{2}{\pi} A_- - C_- \ln^2 s_-$ but from the data of $\Delta\sigma_T$, $\sigma_1 = 0$ seems to be supported aposteriori. See R. Oehme, Phys. Rev. D9, 2695 (1974) for another reason.

[14] Experimental data is from the summary by the Particle Data Group. For σ_T, we have additional data from the ISR (Ref. 2) and the Fermilab (Ref. 1).

TABLE CAPTION

Table 1 Parameters used to fit σ_T and χ^2/number of the data points, in the I-model ($C_R = 0$ in Eqs. (7) and (8)) and in the SI-model ($C_R \neq 0$). Note that $C_+ \cong .5$ mb for all cases, much lower than $\pi/m_\pi^2 = 60$ mb .

FIGURE CAPTIONS

Fig. 1 Fits to σ_T . The data on $\sigma_T^{\pi p}$ for $30 < p_L < 60$ GeV/c are excluded.

Fig. 2 Fits to $\Delta\sigma_T$.

Fig. 3 $(d\sigma/dt)_0$ for $\pi^- p \rightarrow \pi^0 n$ as fitted by the two models.

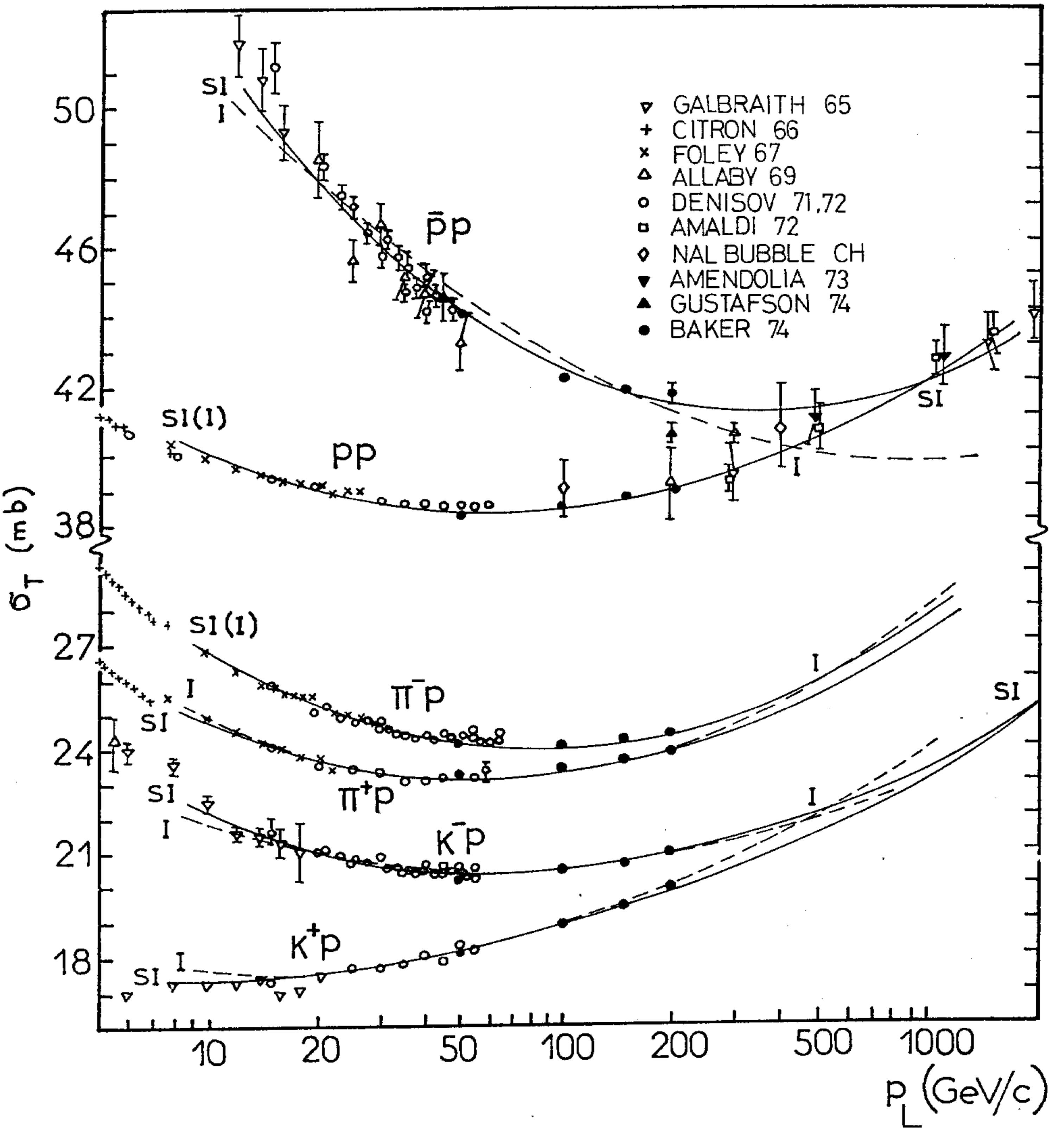
Fig. 4 α = ReF(s)/ImF(s) for pp and $\bar{p}p$.

Fig. 5 α for πp .

Fig. 6 α for Kp . A similar difficulty as in $\pi^- p$ exists for $K^- p$.

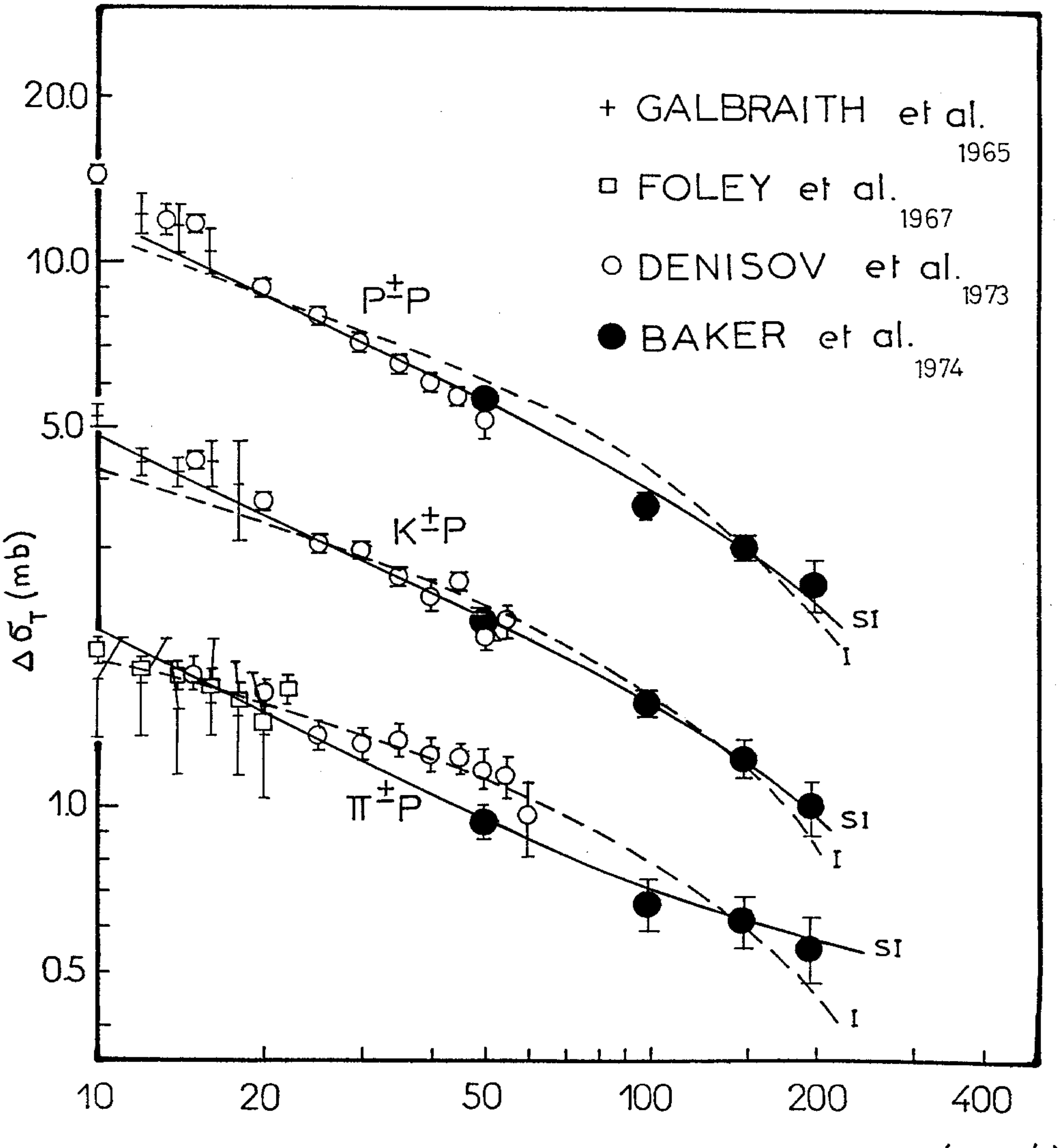
TABLE 1

Parameters and χ^2	I_{pp}	SI_{pp}	I_{Kp}	SI_{Kp}	$I_{\pi p}$	$SI_{\pi p}$
σ_0(mb)	39.93	38.95	19.16	17.28	23.46	22.94
C_+(mb)	0.53	0.50	0.38	0.24	0.63	0.54
$s_+(\mathrm{GeV})^2$	508.8	193.8	61.9	14.6	140.3	111.6
C_-(mb)	0.45	0.14	0.17	0.013	0.076	-0.027
$s_-(\mathrm{GeV})^2$	820.7	551.	794.7	73.3	984.5	500.2
C_R(mbGeV)	-	19.85	-	10.93	-	6.01
χ^2/No. of data points	121/58	58/58	37/35	21/35	37/59	28/59



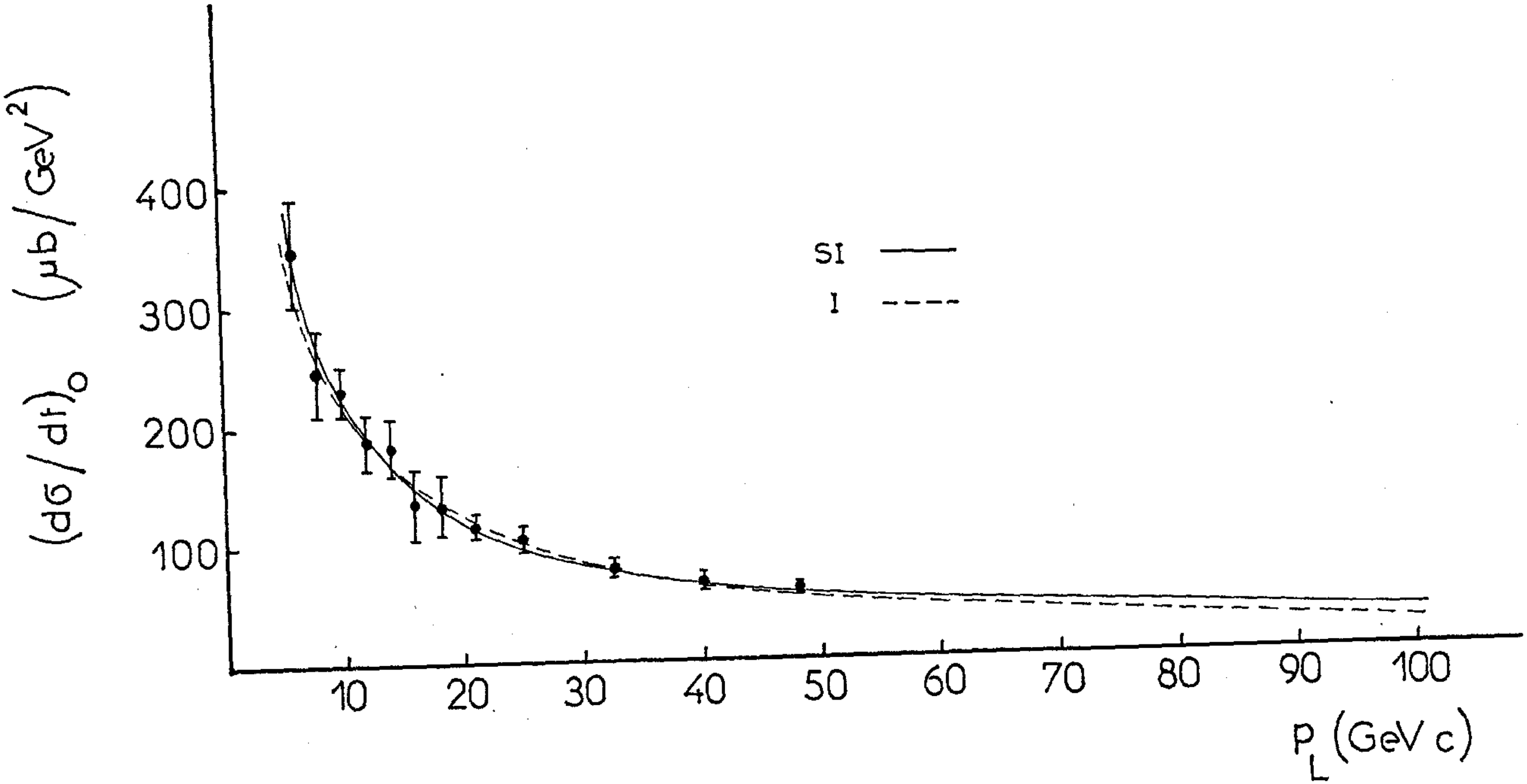
GALBRAITH 65
CITRON 66
FOLEY 67
ALLABY 69
DENISOV 71,72
AMALDI 72
NAL BUBBLE CH
AMENDOLIA 73
GUSTAFSON 74
BAKER 74
p̄p
pp
π⁻p
π⁺p
K⁻p
K⁺p
SI
I
SI(I)
σ_T (mb)
p_L (GeV/c)
50
46
42
38
27
24
21
18
10
20
50
100
200
500
1000

Fig. 1



+ GALBRAITH et al. 1965
FOLEY et al. 1967
DENISOV et al. 1973
BAKER et al. 1974
P±P
K±P
π±P
SI
I
20.0
10.0
5.0
1.0
0.5
$\Delta\sigma_T$ (mb)
10
20
50
100
200
400
p_L (GeV/c)

Fig. 2



$(d\sigma/dt)_o$ $(\mu b/GeV^2)$
400
300
200
100
SI
I
10
20
30
40
50
60
70
80
90
100
P_L (GeV c)

Fig. 3

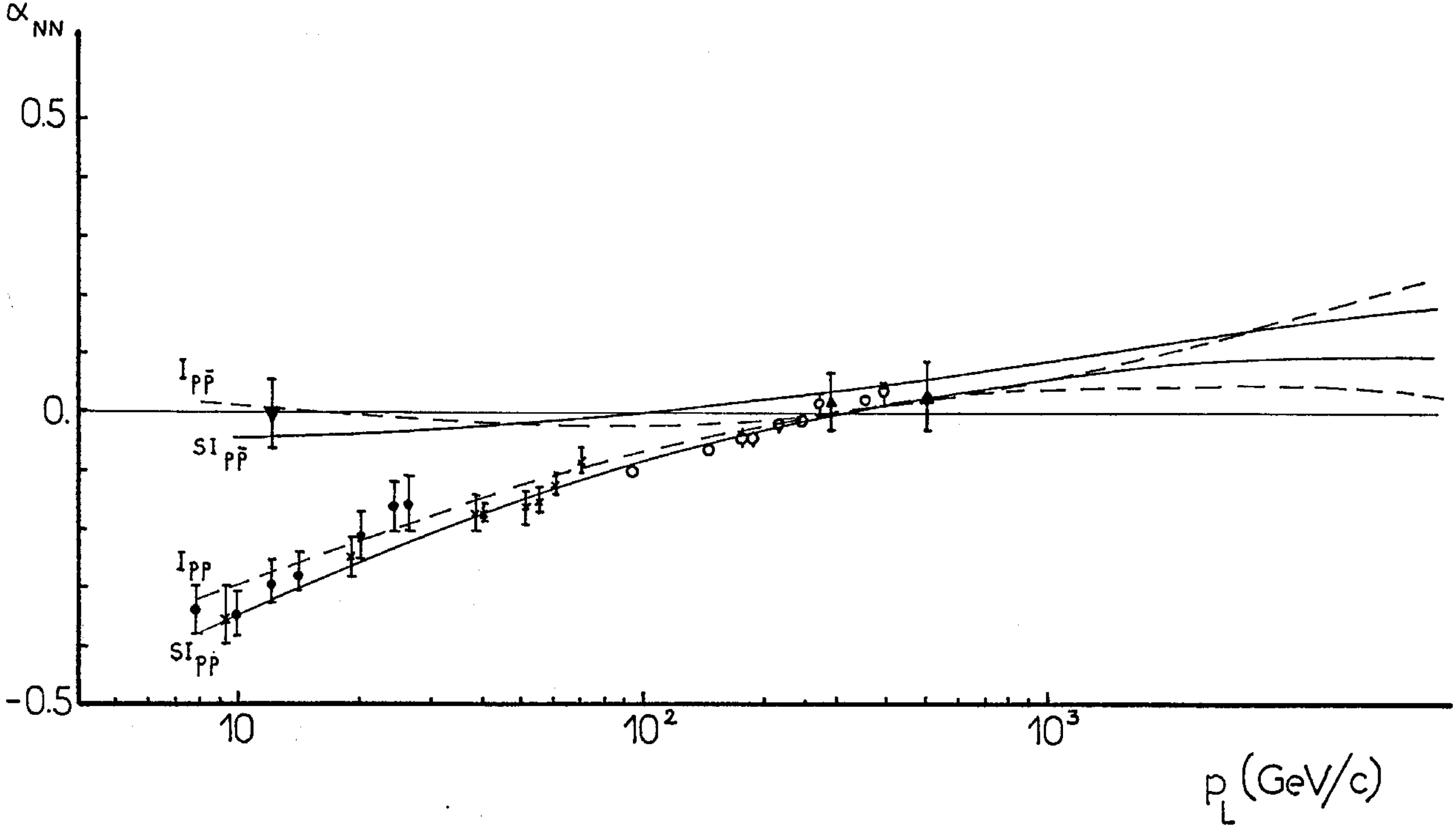


Fig. 4

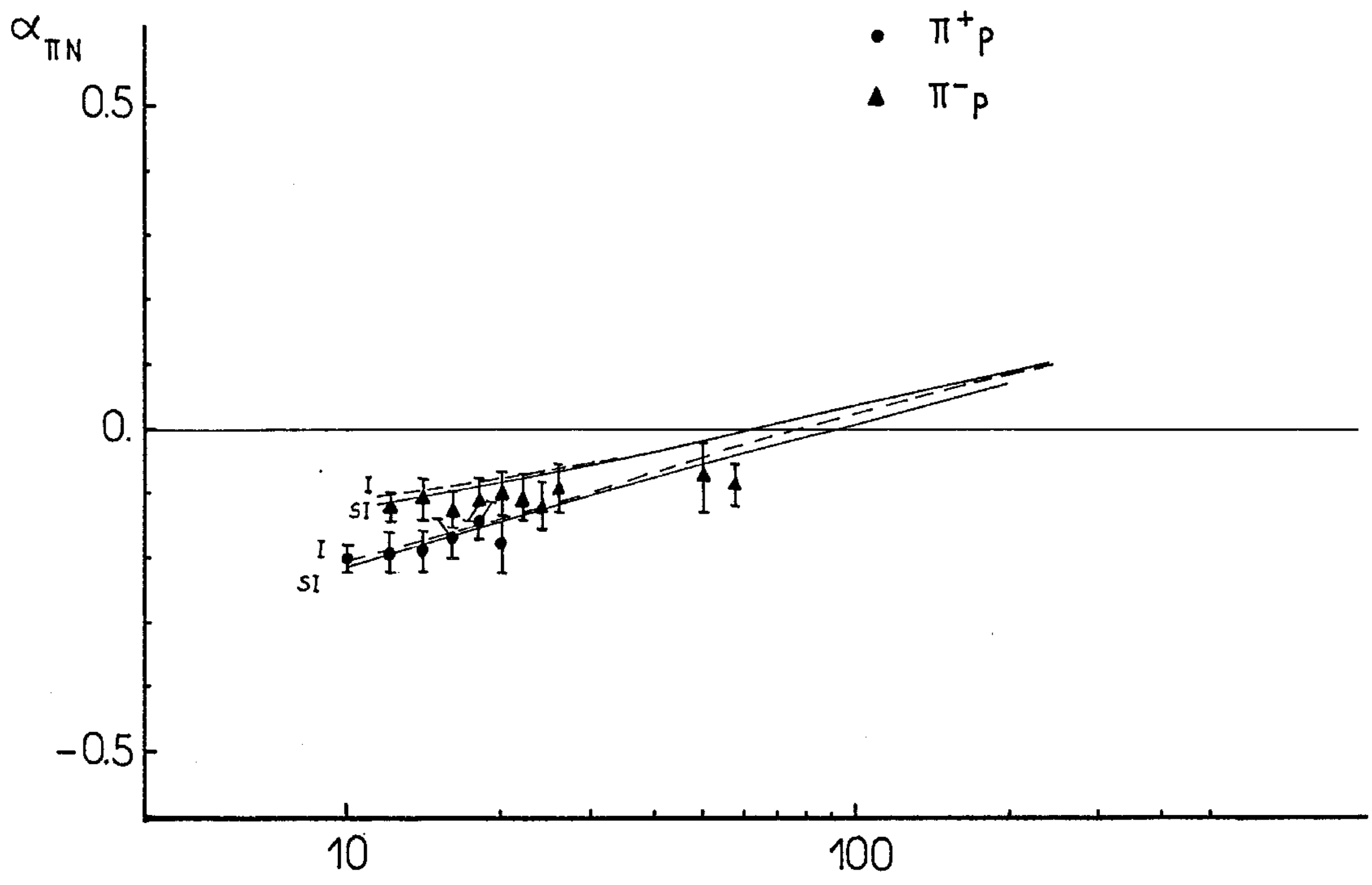


Fig. 5

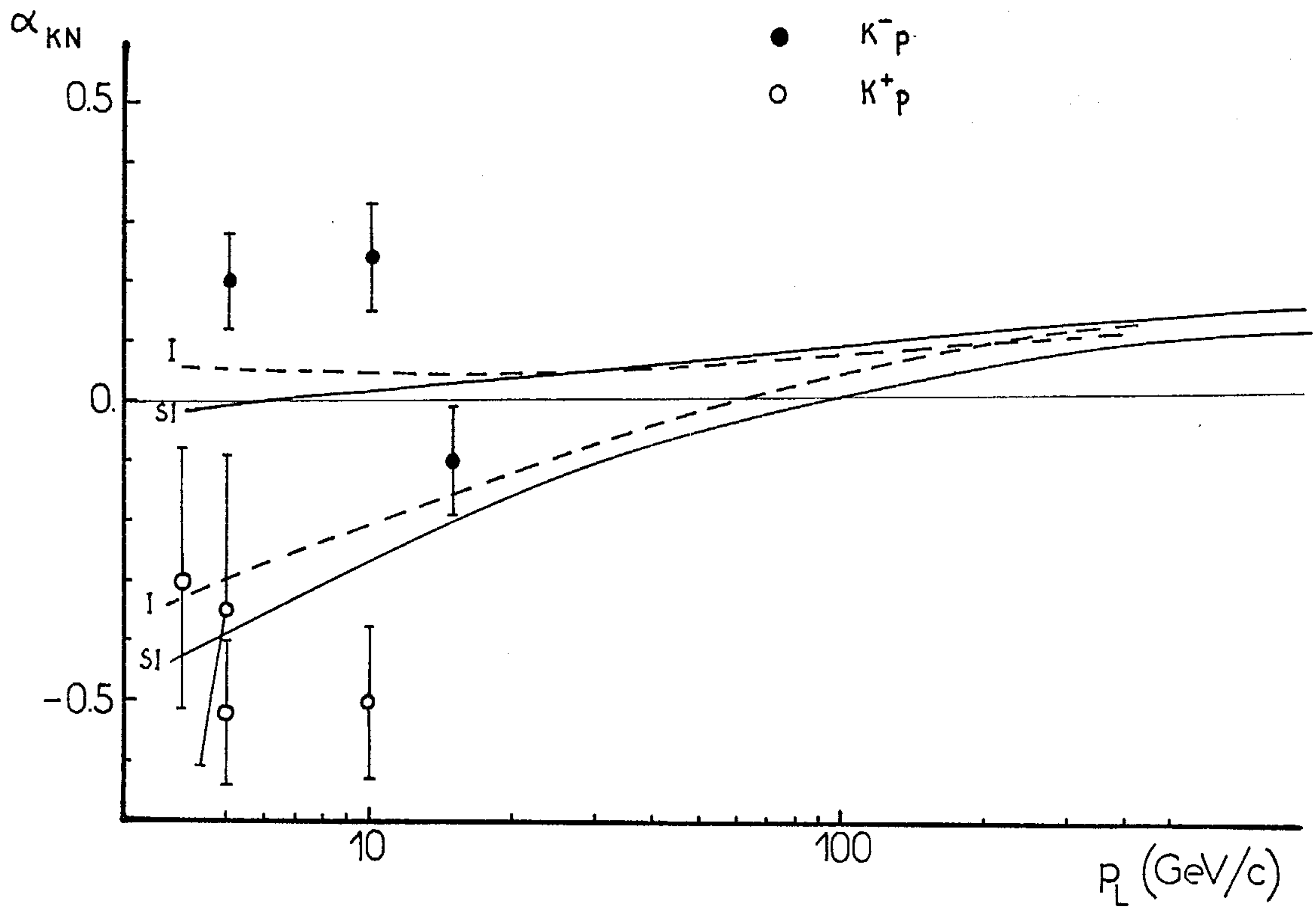


Fig. 6